NOTES: Exam is 30% of final grade
Open everything
Answer all questions
Each question worth 25 points

1) Consider a homogeneous wall, 20 cm thick, constructed of common brick. The wall is initially at a uniform temperature of 20 C. At time t = 0, the outside surface temperture undergoes a step change to 30°C. The temperature remains at 30°C for two hours and then undergoes another step change back to 20°C, where it remains for a long time. The indoor surface temperature is constant at 20 C. Using Duhamel's theorem and the response of a plane wall to a step change in surface temperature, develop an expression for the temperature at the inside surface as a function of time for all times greater than t = 0. The solution should give the temperature in °C as a function of time in seconds. You don't have to actually solve the equation, but you should clearly specify the physical value and engineering units of each variable in the equations and ensure dimensional consistency.

The properties of common brick are as follows.

Density	ρ	= 1600 kg/m^3
Conductivity	k	= 0.69 W/m C
Specific Heat	c_p	= 0.84 kJ/kg C
Thermal Diffusivity	α	= 5.2 x 10^{-7} m^2/s
	L	= 0.20 m

Define $u = \dfrac{T - T_0}{T_\infty - T_0}$ where $T_0 = 20°C$ $T_\infty = 30°C$

$\theta = \dfrac{\alpha t}{L^2}$ x [=] meters
t [=] seconds

Response of plane wall to step in surface temp:

$$U(x,\theta) = 1 - \frac{x}{L} - 2\sum_{n=1}^{\infty} \frac{\sin n\pi\, x/L}{n\pi} e^{-n^2\pi^2\theta}$$

For $t < 2$ hours

$$T = T_0 + (T_\infty - T_0)\left[1 - \left(\frac{x}{L}\right) - 2\sum_{n=1}^{\infty} \frac{\sin n\pi\left(\frac{x}{L}\right)}{n\pi} e^{-(n^2\pi^2\alpha/L^2)t}\right]$$

NOTE THAT @ x = L T = 20°C DUE TO BOUNDARY CONDITIONS.

For $t > 2$ hours

$$u(x,\theta) = U(x,\theta) - U(x,\theta-\theta_1)$$

where $\theta_1 = \frac{\alpha}{L^2}(7200\ s) = 0.0936$

$$u(x,\theta) = 1 - \frac{x}{L} - 2\sum_{n=1}^{\infty} \frac{\sin(n\pi x/L)}{n\pi} e^{-n^2\pi^2\theta}$$

$$- \left[1 - \frac{x}{L} - 2\sum_{n=1}^{\infty} \frac{\sin n\pi x/L}{n\pi} e^{-n^2\pi^2(\theta-\theta_1)} \right]$$

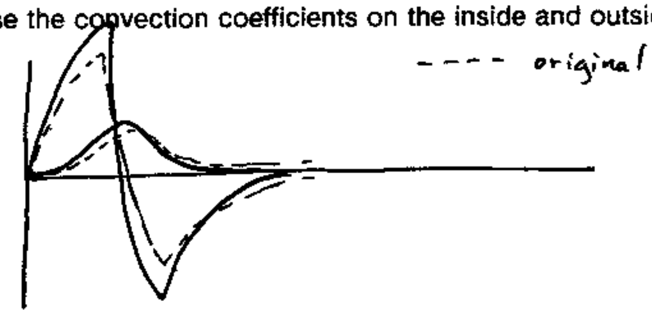
$$= 2\sum_{n=1}^{\infty} \frac{\sin n\pi x/L}{n\pi} e^{-n^2\pi^2\theta} \left(e^{-n^2\pi^2\theta_1} - 1 \right)$$

$$T = T_0 + (T_\infty - T_0)\left[2\sum_{n=1}^{\infty} \frac{\sin n\pi x/L}{n\pi} e^{-(n^2\pi^2\alpha/L^2)t} \left(e^{-n^2\pi^2\frac{\alpha}{L^2}(7200)} - 1 \right) \right]$$

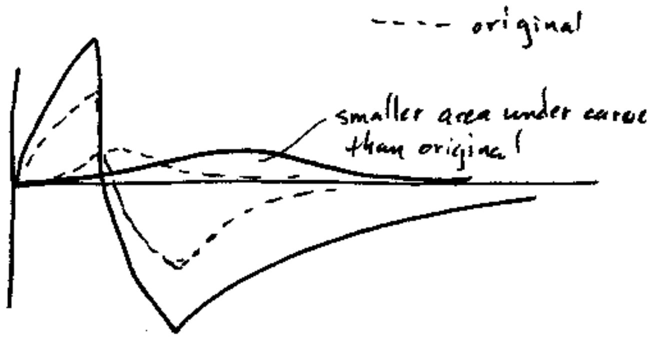
NOTE THAT AT $x = L$, $T = 20°C$

2) Figure A shows the heat flux at the indoor and outdoor surface for a 4" insulated frame wall exposed to a 1°C triangular pulse in outdoor temperature with a base of 2 hours. Qualitatively describe the effects on the following changes in wall characteristics.

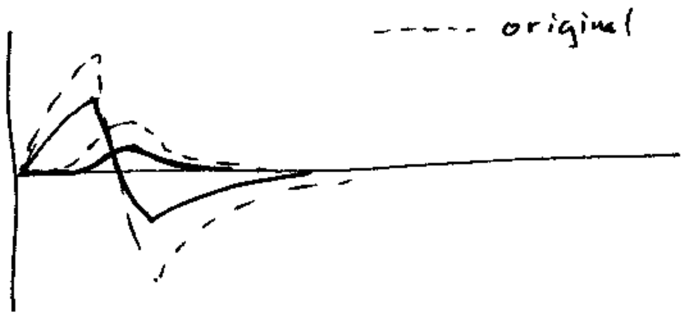
a) Increase the convection coefficients on the inside and outside surfaces.



b) Add a layer of 2 inch face brick on the outside surface.



c) Replace the insulation with material having a lower thermal conductivity.



3) Figure A shows the heat flux at the indoor and outdoor surface for a 4" insulated frame wall exposed to a 1°C triangular pulse in outdoor temperature with a base of 2 hours.

a) Identify the Y_j and Z_j response factors on the figure. — SEE FIGURE A.

b) The wall, initially at 20°C for t < 0, is exposed to temperature boundary conditions given by the table below. Complete the table by calculating the heat flux.

Time (hr)	Outdoor Temperature (°C)	Indoor Temperature (°C)	Indoor Heat Flux (W/m²)
0	20.0	20.0	0.0
1	30.0	20.0	0.6
2	30.0	20.0	1.7
3	20.0	20.0	1.3

a) $Y_0 = 0.06$ $\quad Z_0 = 0.47$ $\quad$ FOR SYMMETRIC WALL

$Y_1 = 0.11$ $\quad Z_1 = -0.26$ $\quad X_j = Z_j$

$Y_2 = 0.02$ $\quad Z_2 = -0.02$

b) $$q_i(t) = \sum_{j=0}^{\infty} Y_j\, T_o(t - j\Delta) - \sum_{j=0}^{\infty} Z_j\, T_i(t - j\Delta)$$

Define $T^* = T - 20°C$

$$q_i(t) = \sum_{j=0}^{\infty} Y_j\, T_o^*(t - j\Delta)$$

$$q_i(1) = Y_0 \cdot [T_o(1) - 20°C] = 0.06 \cdot 10 = \underline{0.6\ W/m^2}$$

$$q_i(2) = Y_0 \cdot [T_o(2) - 20°C] + Y_1 [T_o(1) - 20°C]$$

$$= 0.06 \cdot 10 + 0.11 \cdot 10 = \underline{1.7\ W/m^2}$$

$$q_i(3) = Y_0 \cdot [T_o(3) - 20°C] + Y_1 [T_o(2) - 20°C] + Y_2 \cdot [T_o(1) - 20°C]$$

$$= 0.06 \cdot 0 + 0.11 \cdot 10 + 0.02 \cdot 10 = \underline{1.3\ W/m^2}$$

4. The heat transfer through a lightweight concrete wall is being analyzed by finite difference methods. At some time $t=t_0$, the following temperatures are observed in the wall. (Values of x are given in meters.)

$$T(x=0,t_0) = 24°C \quad T(x=0.02,t_0) = 20.8°C \quad T(x=0.04,t_0) = 22°C \quad T(x=0.06,t_0) = 24°C$$

If the temperatures at x=0 and x=0.06 are maintained at 24°C, what are the temperatures at the two interior locations at $t=t_0+5$ minutes?

The properties of the wall are as follows.

Density	ρ = 1800 kg/m³
Conductivity	k = 1.70 W/m°C
Specific Heat	c_p = 0.92 kJ/kg C

$$\alpha = \frac{k}{\rho c_p} = \frac{1.7}{(1800)(920)} = 1.0266 \times 10^{-6} \ m^2/s$$

Consider Euler finite difference method. For interior nodes

$$T_m^{p+1} = F_o\left(T_{m-1}^p + T_{m+1}^p\right) + (1-2F_o)\,T_m^p$$

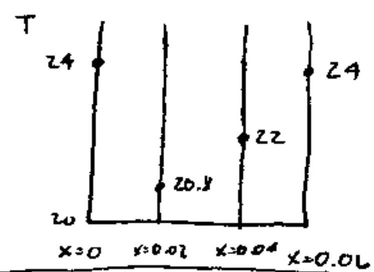
$$F_o = \frac{\alpha \Delta t}{\Delta x^2} < 1/2 \quad \text{FOR STABILITY}$$

$$\Delta t < \frac{\Delta x^2}{2\alpha} \qquad \Delta x = 0.02 \ m$$

$$\Delta t < \frac{0.0004}{2(1.0266\times10^{-6})} < 194.8 \text{ seconds}$$

LET $\underline{\Delta t = 150 \ s}$ = 2 1/2 minutes $\quad F_o = 0.385$

Define



m =	0	1	2	3

$p = 0$ @ $t = t_0$

@ $t = t_0 + 150$ s

$$T_1' = F_o(T_0^\circ + T_2^\circ) + (1-2F_o)T_1^\circ = \underline{22.49}\ °C$$

$$T_2' = F_o(T_1^\circ + T_3^\circ) + (1-2F_o)T_2^\circ = \underline{22.31}\ °C$$

@ $t = t_o + 300$s

$$T_1^2 = F_o(T_0' + T_2') + (1-2F_o)T_1' = \underline{23.00}\ °C$$

$$T_2' = F_o(T_1' + T_3') + (1-2F_o)T_2' = \underline{23.03}\ °C$$

HEAT FLUX AT WALL SURFACE
Unit Outdoor Temperature Pulse

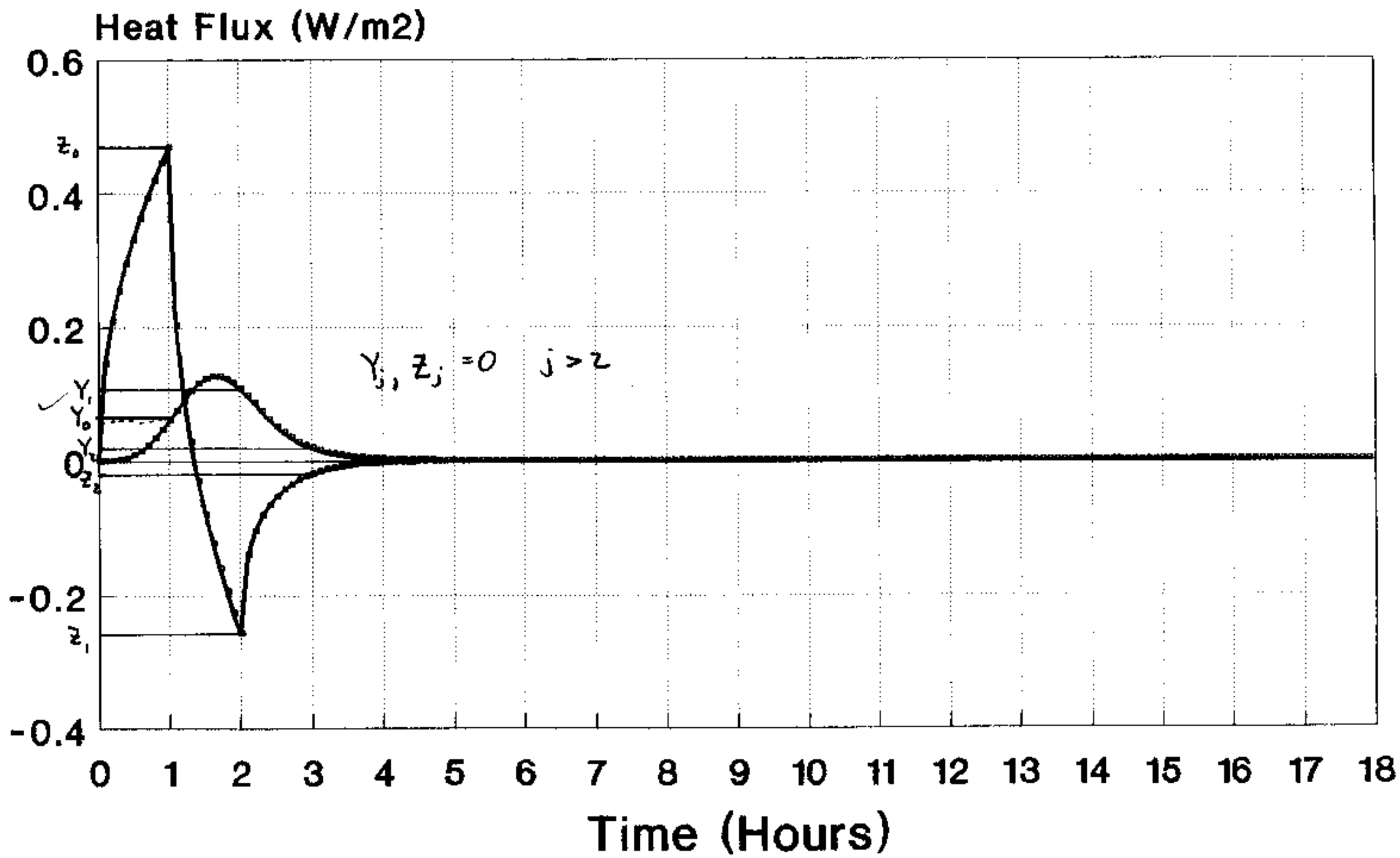


FIGURE A